Section 3.3 - Rules for Differentiation

Rule 1

Derivative of a Constant Function

If f is the function with the constant value c, then

$$\frac{df}{dx} = \frac{d}{dx}(c) = 0$$

Proof of Rule 1

Find the derivative of $f(x) = x^3$

Rule 2

Power Rule for Positive Integer Powers of x

Or just the POWER RULE

If n is a positive integer, then

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Find the derivate of $4x^2$

a) $f(x) = x^5$

b) $g(x) = \sqrt[5]{x}$

c) $h(x) = \frac{1}{x^5}$

Rule 3

The Constant Multiple Rule

If u is a differentiable function of x and c, then

$$\frac{d}{dx}(cu) = c\frac{du}{dx}$$

In ENGLISH

- If a differentiable function is multiplied by a constant, then its derivative is multiplied by the same constant

a) $y = \dfrac{3}{\sqrt[4]{x}}$

b) $y = \dfrac{3}{2x^2}$

Rule 4

The Sum Difference Rule

If u and v are differentiable functions of x, then their sum and difference are differentiable at every point where u and v are differentiable.

$$\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x)$$

Example

Find $\frac{dp}{dt}$ if $p = t^3 + 6t^2 - \frac{5}{3}t + 16$

a) $f(x) = -4x^3 - 5x^2 + 4x - 2$

b) $h(x) = -4\sqrt{x^7} - 5\sqrt[5]{x^6}$

c) $y = \frac{-1}{2}\frac{-4x^2 - 4x + 3}{x^2}$

Does the curve $y = x^4 - 2x^2 + 2$ have any horizontal tangents? If so, where?

Using the window [-10,10] by [-10,10], the graph of $y=0.2x^4 - 0.7x^3 - 2x^2 + 5x + 4$ has three horizontal tangents. At what points do these horizontal tangents occur?

Rule 5

The Product Rule

The product of two differentiable functions u and v is differentiable, and

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

OR

$$\frac{d}{dx} f(x)*g(x) = f'(x)*g(x) + g'(x)*f(x)$$

Example

Find $f'(x)$ if $f(x) = (x^2 + 1)(x^3 + 3)$

Rule 6

The Quotient Rule

At a point where $v \neq 0$, the quotient $y = u/v$ of two differentiable functions is differentiable, and

$$\frac{d}{dx}(u/v) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

OR

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x) * f'(x) - g'(x)*f(x)}{(g(x))^2}$$

Differentiate

$$f(x) = \frac{x^2 - 1}{x^2 + 1}$$

Let $y = uv$ be the product of the functions u and v. Find $y'(2)$ if

$u(2) = 3$, $u'(2) = 4$, $v(2) = 1$, and $v'(2) = 2$

Rule 7

Power Rule for Negative Integer Powers of x

If n is a negative integer and $x \neq 0$, then

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Examples

Find the derivatives of each

1.) $f(x) = 3x^3 - x^2 + 4x^{-1} - 2x^{-3}$

2.) $f(x) = 5\sqrt{x}$

3.) $f(x) = -3x\sqrt{x}$

4.) $f(x) = (x^3 - 2)(x + 1)$

5.) $f(x) = \dfrac{x - 3}{x^2 + 9}$

$$f(x) = \frac{\sqrt{x}}{3 + x^{-2}} = \frac{x^{\frac{1}{2}}}{3+x^{-2}}$$

$$f'(x) = \frac{\frac{1}{2}x^{-\frac{1}{2}}\left(3+x^{-2}\right) + \left(-2x^{-3}\right)\left(x^{\frac{1}{2}}\right)}{\left(3+x^{-2}\right)^2}$$

$$= \frac{\frac{3}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{5}{2}} + 2x^{-\frac{5}{2}}}{\left(3+x^{-2}\right)^2}$$

$$= \frac{\frac{3}{2}x^{-\frac{1}{2}} + \frac{5}{2}x^{-\frac{5}{2}}}{\left(3+x^{-2}\right)^2} = \frac{\frac{3}{2\sqrt{x}} + \frac{5}{2\sqrt{x^5}}}{\left(3+x^{-2}\right)^2}$$

Finding Higher Order Derivatives

$$f'(x)$$

$$f''(x)$$

$$f'''(x)$$

$$f^{(4)}(x)$$

Find the first four derivatives of $y = x^3 - 5x^2 + 2$

$y' = 3x^2 - 10x$

$y'' = 6x - 10$

$y''' = 6$

$y^{(4)} = 0$ OR $y^{(iv)} = 0$

Find the first five derivatives of

$f(x) = 5x^4 - 2x^3 + x^2 + 7$

$f'(x) = 20x^3 - 6x^2 + 2x$

$f''(x) = 60x^2 - 12x + 2$

$f'''(x) = 120x - 12$

$f^{(4)}(x) = 120$

$f^{(5)}(x) = 0$

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